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SUMMARY

This report presents a robust control design using strictly positive realness for second-order dynamic systems. The robust strictly positive real controller allows the system to be stabilized with only acceleration measurements. An important property of this design is that stabilization of the system is independent of the system parameters. The control design connects a virtual system to the given plant. The combined system is positive real regardless of system parameter uncertainty. Then any strictly positive real controllers can be used to achieve robust stability. A spring-mass system example and its computer simulations are presented to demonstrate this controller design.

1. INTRODUCTION

Positive real (PR) systems have many applications for shape and vibration control of large flexible structures. In most of those PR designs, the output of the plant is usually assumed to include velocity, and the sensors are assume to be collocated with the actuators. In [1], position and velocity feedback are used together to control large space structures, and the controllers are strictly positive real. PR feedback with velocity measurement is examined in [2] for the control of a flutter mode. [3] presents a robust multivariable control of structures using a passive controller in which the velocity sensors are collocated with the control actuators. Several passive control designs using acceleration, velocity and position measurements are presented in [4]. [5] generalizes the designs in [4] to handle nonlinear systems. The method presented in [6] uses displacement sensors. Similarly, [7] examines direct position plus velocity feedback. A feedforward positive real design can be seen from [11].

Nevertheless, in some application areas, only acceleration is directly measurable. Even though velocity and position may be obtained by integrating the measured acceleration, bias in velocity and position will decrease the accuracy of the integration. Therefore, in this study we develop a robust controller for multivariable second-order system when only acceleration is directly measurable.

In this report, we review some definitions and a theorem associated with dissipativeness and passivity. Dissipativeness and passivity are then related to strictly positive realness and positive realness. Using these backgrounds we develop a virtual system to compute an output which will make the combined system of the plant and the virtual system positive real (PR). The inputs to the virtual system are only acceleration and the control force applied to the plant. More important, the virtual system is model independent and thus the global system is robustly positive real. Therefore the input / output controller can be constructed by any strictly positive real controllers. When the stiffness matrix of the second-order system is positive definite, we show that it is possible to stabilize the displacement if the actuators are properly located. With this design, the displacement is globally asymptotically stable. A spring-mass example with three masses and no damping is used to illustrate our design method. Computer simulations are also presented.

2. PRELIMINARIES

The concept of dissipativeness describes an important input-ouput property of dynamical systems. Consider a system with input u and output y, where u is an $m\times 1$ vector and y is a $p\times 1$ vector. A supply rate for the system is defined as follows.

Definition 1 [8]: A supply rate is a real function of u and y defined as

$$w(u, y) = y^T Q\, y + 2\, y^T S\, u + u^T R\, u \tag{1}$$

where Q, S, and R are constant real matrices with dimensions $p\times p$, $p\times m$ and $m\times m$ respectively.

Q and R are usually symmetric matrices. w(u,y) is often called the input energy into the system. Dissipativeness is defined with respect to the supply rate w(u,y) in the following definition.

Definition 2 [8]: The system with input u and output y is called dissipative with respect to the supply rate w(u,y) if for all locally integrable u(t) and all $T\geq t_0$, we have

$$\int_{t_0}^{T} w(t)\,dt \geq 0 \tag{2}$$

where $x(t_0)=0$ and w(t)=w(u(t),y(t)) is evaluated along the trajectory of the system interested.

Eq.(2) means that an initially unexcited system can only absorb energy as long as the system is dissipative. If the supply rate represents the input energy into the system, then Eq.(2) states that a system with no initially stored energy transforms the input energy into either stored energy or dissipated energy. Thus no energy can be generated from a dissipative system.

Passivity is defined as a special case of dissipativeness.

Definition 3 [8]: A system is passive if and only if it is dissipative with respect to the supply rate

$$w(u, y) = u^T y \tag{3}$$

An algebraic condition for passivity can be found if the system is represented by the state-space equations

$$\begin{aligned} \dot{x} &= f(x) + G(x)u \\ y &= h(x) + J(x)u \end{aligned} \tag{4}$$

where f(x) and h(x) are real vector functions of the state vector x, with f(0)=0, h(0)=0, and G(x) and J(x) are real matrix functions of x. These four functions are assumed to be infinitely differentiable. We also assume that u and y have the same dimension. The system is furthermore assumed to be completely controllable. Theorem 1 provides a test for the passivity of a system written in the form of Eq. (4).

Theorem 1 [9]: The system is passive if and only if there exist real functions $\phi(x)$, $l(x)$ and $W(x)$ with $\phi(x)$ continuous and with

$$\phi(x) \geq 0, \quad \text{for all } x \tag{5}$$

and

$$\phi(0)=0 \tag{6}$$

such that

$$
\begin{aligned}
&\text{(i)} \quad \nabla^T \phi(x)\, f(x) = -\, l^T(x)\, l(x) \\
&\text{(ii)} \quad 1/2\, G^T(x) \nabla \phi(x) = h(x) - W^T(x)\, l(x) \\
&\text{(iii)} \quad J(x) + J^T(x) = W(x)^T W(x)
\end{aligned}
\tag{7}
$$

Moreover, if J (x) is a constant matrix, then W(x) may be taken to be constant.

The function $\phi(x)$ is generally not unique for a given dynamic passive system. Nevertheless, a physical meaning can be attached to it. It is shown in [9] that

$$
2 \int_{t_0}^{T} u^T(t) y(t)\, dt = \phi[x(T)] - \phi[x(t_0)] + \int_{t_0}^{T} [l(x) + W(x)u]^T [l(x) + W(x)u]\, dt \tag{8}
$$

Eq.(8) may be interpreted as the conservation of energy equation. $\frac{1}{2}\phi(x)$ is a stored energy for the system. The first integral corresponds to the input energy to the dynamic system. The second one is proportional to dissipated energy, and it is always nonnegative. As a consequence, Eq.(8) means that the energy input is equal to the variation of stored energy plus the loss of energy which is a positive function.

A linear system is passive if and only if its transfer matrix is positive real [10]. Passivity can thus be seen as a generalization of positive realness for nonlinear systems. Since the systems investigated here are linear, we will equivalently use these two concepts for the rest of this report.

3. A VIRTUAL SYSTEM DESIGN

The multivariable system (Plant (P)) is described by

$$
M\ddot{x} + D\dot{x} + Kx = Bu \tag{9}
$$

where u is an m×1 control vector, x is an n×1 state vector, M is an n×n symmetric positive definite matrix, D and K are n×n symmetric positive semi-definite matrices, and B is an n×m matrix. Let a virtual system (V) be defined with the following equation

$$\ddot{x}_a = \Lambda \ddot{x} + B'u \tag{10}$$

where Λ is an l×n matrix, B' is an l×m matrix, and x_a is an l×1 vector. The following Theorem 2 allows us to compute an output y that makes the global system (which is a combined system of the given plant and the virtual system) positive real.

Theorem 2: Let H_v, Λ and B' be chosen such that

$$\begin{aligned} 2H_v\Lambda &= B^T \\ B'^T M_a^T &= 2H_v \end{aligned} \tag{11}$$

where M_a is an $l \times l$ positive semi-definite matrix. If

$$y = H_v \dot{x}_a \tag{12}$$

then the system with input u and output y is positive real.

This scheme is illustrated in Fig. 1

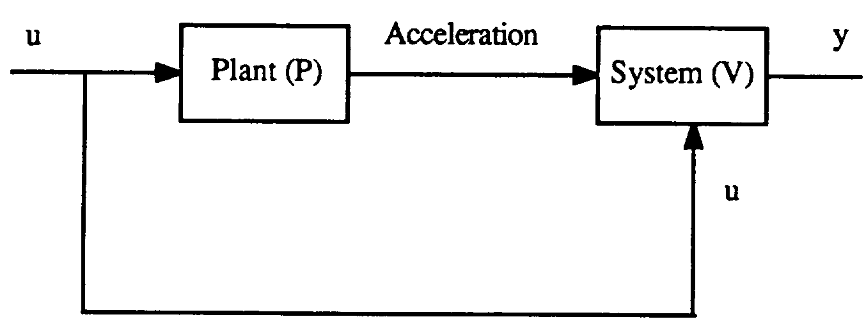

Figure 1. A Virtual System

Proof: For this proof, it is useful to represent the system with a state-space representation. Let

$$X^T = (x_1^T\, x_2^T\, x_3^T\, x_4^T) = (x^T\, \dot{x}^T\, x_a^T\, \dot{x}_a^T) \tag{13}$$

The equations describing the global system may be rewritten as

$$\begin{cases} \dot{X} = f(X) + G(X)u \\ Y = h(X) + J(X)u \end{cases} \tag{14}$$

where

$$f(X) = \begin{pmatrix} x_2 \\ -M^{-1}Dx_2 - M^{-1}Kx_1 \\ x_3 \\ -\Lambda M^{-1}Dx_2 - \Lambda M^{-1}Kx_1 \end{pmatrix} \tag{15}$$

$$G(X) = \begin{pmatrix} 0 \\ M^{-1}B \\ 0 \\ \Lambda M^{-1}B + B' \end{pmatrix} \tag{16}$$

$$h(X) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ H_v x_4 \end{pmatrix} \tag{17}$$

$$J(X)=0 \tag{18}$$

Let a candidate for the function $\phi(X)$ in Theorem 2 be

$$\phi(X) = \frac{1}{2}\dot{x}^T M \dot{x} + \frac{1}{2}x^T K x + \frac{1}{2}(\dot{x}_a - \Lambda\dot{x})^T M_a (\dot{x}_a - \Lambda\dot{x}) \tag{19}$$

where M_a is positive semi-definite. The sum of the first two terms corresponds to the stored energy of the plant. The additional term is added for the positive real design. The function $\phi(X)$ can be written using the state variables as

$$\phi(X) = \frac{1}{2}x_2^T M x_2 + \frac{1}{2}x_1^T K x_1 + \frac{1}{2}(x_4 - \Lambda x_2)^T M_a (x_4 - \Lambda x_2) \qquad (20)$$

$\phi(X)$ is a positive function and $\phi(0)=0$. It must be checked that there exists a function $l(X)$ such that

$$\nabla^T \phi(X)\, f(X) = -l^T(X)\, l(X) \qquad (21)$$

This calculation is considerably simplified when we notice that

$$\nabla^T \phi(X)\, f(X) = \left. \frac{d\phi(X)}{dt} \right|_{u=0} \qquad (22)$$

As a consequence we have

$$\begin{aligned} \nabla^T \phi(X) f(X) = \dot{x}^T (M\ddot{x} + Kx) &+ \frac{1}{2}(\ddot{x}_a - \Lambda\ddot{x})^T M_a (\dot{x}_a - \Lambda\dot{x}) \\ &\left. + \frac{1}{2}(\dot{x}_a - \Lambda\dot{x})^T M_a (\ddot{x}_a - \Lambda\ddot{x}) \right|_{u=0} \end{aligned} \qquad (23)$$

When $u = 0$, the last two terms cancel out and therefore

$$\nabla^T \phi(X)\, f(X) = \left. \dot{x}^T (M\ddot{x} + Kx) \right|_{u=0} \qquad (24)$$

Thus we finally have

$$\nabla^T \phi(X)\, f(X) = -\dot{x}^T D \dot{x} = -x_2^T D x_2 \qquad (25)$$

Since D is positive semi-definite, it is possible to find a matrix R such that $D = R^T R$. The above equality becomes

$$\nabla^T \phi(X)\, f(X) = -(R x_2)^T (R x_2) = -l(X)^T l(X) \qquad (26)$$

where $l(X) = R x_2$. Thus equality (i) from Theorem 1 is satisfied. Equality (iii) of Eq. (7)

reduces to

$$J(X) + J^T(X) = W^T(X)\, W(X) = 0 \tag{27}$$

The function W(X) is therefore equal to zero. Equality (ii) of Eq. (7) becomes

$$h(X) = \frac{1}{2} G^T(X)\, \nabla\phi(X) \tag{28}$$

Only the partial derivatives with respect to velocity will be used to evaluate Eq. (28). We have

$$\begin{aligned} \frac{\partial\phi}{\partial x_2} &= x_2^T M + (\Lambda x_2 - x_4)^T M_a \Lambda \\ \frac{\partial\phi}{\partial x_4} &= (x_4 - \Lambda x_2)^T M_a \end{aligned} \tag{29}$$

The function h(X) is such that

$$2h(X) = (M^{-1} B)^T \left(\frac{\partial\phi}{\partial x_2}\right)^T + (\Lambda M^{-1} B + B')^T \left(\frac{\partial\phi}{\partial x_4}\right)^T \tag{30}$$

Obvious simplifications yield

$$2h(X) = (B^T - B'^T M_a^T \Lambda)\, x_2 + B'^T M_a^T x_4 \tag{31}$$

h(X) equals $H_v x_4$ if the following equations are satisfied

$$\begin{aligned} B^T - B'^T M_a^T \Lambda &= 0 \\ B'^T M_a^T &= 2H_v \end{aligned} \tag{32}$$

Those equations can be rewritten as

$$
\begin{aligned}
2H_v\Lambda &= B^T \\
B'^T M_a^T &= 2H_v
\end{aligned}
\tag{33}
$$

and the theorem is proved •

There are several possible ways to solve the above system of equations. Given H_v and B, we can solve for some possible Λ, M_a and B'. At the end of the calculation, it must be checked that M_a is positive semi-definite. Another method consists of choosing B, Λ and a positive semi-definite M_a and then solving for possible B' and H_v.

4. CHOICE OF A CONTROLLER

If the output of the global system is chosen as in Theorem 2, then the system is positive real. Thus the closed-loop system is uniformly asymptotically stable with zero input if the controller is strictly positive real [3]. That is, for this case, we have

$$
\lim_{t \to \infty} (H_v \dot{x}_a) = 0 \tag{34}
$$

Our goal is to let x go to zero. Theorem 3 may be used to achieve this goal.

Theorem 3: Assume that Theorem 2 is used to make the global system PR. Furthermore assume that

(i) $B^T \ddot{x} = 0$ and $u = 0$ imply $\ddot{x} = 0$.

(ii) K is positive definite.

(iii) The system is connected to a dissipative closed-loop controller.

Then $\lim_{t \to \infty} x(t) = 0$.

Fig.2 shows the control scheme for the plant (P).

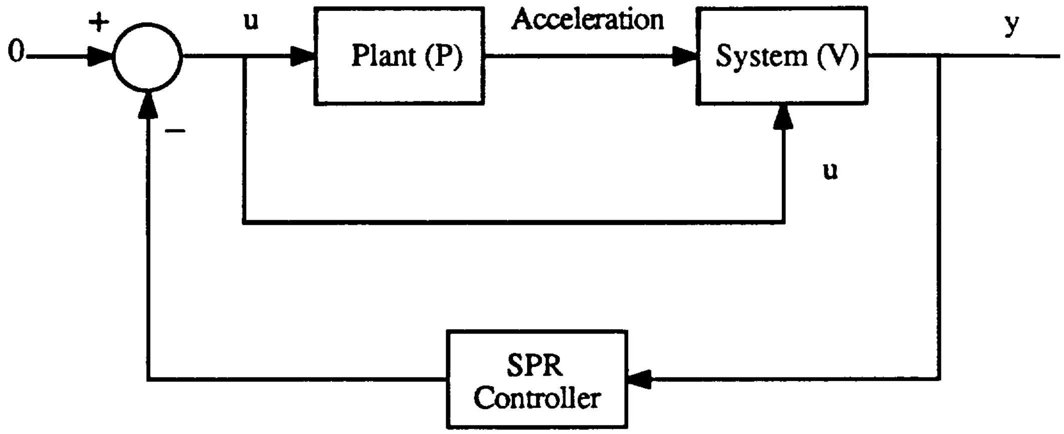

Figure 2. A SPR Controller for the Virtual System

Theorem 3 allows us to design a robust controller for Plant (P). No knowledge of the constant matrices M, D and K is required. Furthermore, the only measurements needed are acceleration and input. Acceleration may easily be measured for many practical systems by using common accelerometers. The input u may be obtained by measuring the output of the SPR controller.

The proof of Theorem 3 uses the following two lemmas.

Lemma 1: Assume that the Laplace transform of f(t) and $\dot{f}(t)$ exist in a neighborhood of the origin. Furthermore assume that $\lim_{t \to \infty} f(t) = 0$. Then $\lim_{t \to \infty} \dot{f}(t) = 0$.

Proof: Let F(s) be the Laplace transform of f(t). The final value theorem yields

$$\lim_{s \to 0} s\,F(s) = 0 \tag{35}$$

The Laplace transform of the derivative of f(t) is

$$L[\dot{f}(t)] = sF(s) - f(0) \tag{36}$$

As a consequence we have

$$\lim_{t \to \infty} \dot{f}(t) = \lim_{s \to 0} s\,(sF(s) - f(0)) = 0 \tag{37}$$

Lemma 2: Let $\varepsilon(t)$ be a function of time and let $\varepsilon(t)$ go to zero as time increases. Then if x satisfies the differential equation

$$D\,\dot{x} + K\,x = \varepsilon \tag{38}$$

where D is positive semi-definite and K is positive definite, then x converges to zero.

Proof: Let m denote the rank of D. There exists an invertible n×n matrix P such that

$$D^{*} = P\ D\ P^{-1} \tag{39}$$

where

$$D^{*} = \begin{bmatrix} 0 & 0 \\ 0 & D_{22} \end{bmatrix} \tag{40}$$

D_{22} is an m×m positive definite matrix. Let K^{*} be defined as

$$K^{*} = P\ K\ P^{-1} \tag{41}$$

K^{*} may be written as

$$K^{*} = \begin{bmatrix} K^{*}_{11} & K^{*}_{12} \\ K^{*}_{21} & K^{*}_{22} \end{bmatrix} \tag{42}$$

The dynamical equation can be written as

$$P\,D\,P^{-1}(P\,\dot{x}) + P\,K\,P^{-1}(P\,x) = P\,\varepsilon(t) \tag{43}$$

Let y =Px and $\eta(t) = P\,\varepsilon(t)$. The system is now described by

$$D^{*}\,\dot{y} + K^{*}\,y = \eta(t) \tag{44}$$

If $y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ and $\eta = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$ these equations are reduced to

$$\begin{cases} K^*_{11} y_1 + K^*_{12} y_2 = \eta_1(t) \\ D_{22} \dot{y}_2 + K^*_{21} y_1 + K^*_{22} y_2 = \eta_2(t) \end{cases} \tag{45}$$

The first equation can be solved in terms of y_1 and Eq. (45) reduces to

$$\begin{cases} y_1 = -K^{*-1}_{11} K^*_{12} y_2 + K^{*-1}_{11} \eta_1(t) \\ D_{22} \dot{y}_2 + (K^*_{22} - K^*_{21} K^{*-1}_{11} K^*_{12}) y_2 = \eta_2(t) + K^{*T}_{12} K^{*-1}_{11} \eta_1(t) \end{cases} \tag{46}$$

D_{22} and $(K^*_{22} - K^*_{21} K^{*-1}_{11} K^*_{12})$ are positive definite matrices. Thus y_2 may be considered as the output of a strictly stable system. The output of the strictly stable system converges to zero. The parameter y_2 will therefore go to zero. The first equality in Eq.(46) shows that y_1 also goes to zero. Consequently, y converges to zero and so does x.

Proof of Theorem 3: (-u) is the output of the dissipative controller. A dissipative controller is always strictly stable. Knowing that y goes to zero, we can therefore conclude that u also goes to zero. Furthermore, we have

$$2 H_v \ddot{x}_a = 2 H_v \Lambda \ddot{x} + 2 H_v B \dot{u} \tag{47}$$

by multiplying Eq.(10) with $2 H_v$. Since $2 H_v \Lambda = B^T$, this equation may be rewritten as

$$B^T \ddot{x} = 2 H_v \ddot{x}_a - 2 H_v B \dot{u} \tag{48}$$

$2 H_v B \dot{u}$ goes to zero as u goes to zero. Furthermore, we know that $y = H_v \dot{x}_a$ converges to zero as time increases. Using Lemma 1 allows us to say that $H_v \ddot{x}_a$ also goes to zero if we assume that the Laplace transforms of $\dot{x}_a$ and its derivative exist. As a consequence, $B^T \ddot{x}$ goes to zero. The equations describing the system are linear and consequently continuous.

Thus, if $B^T \ddot{x}$ and u go to zero, $\ddot{x}$ goes to zero according to assumption (i) in Theorem 3. The dynamics of the closed-loop system is now

$$D\dot{x} + Kx = Bu - M\ddot{x} = \varepsilon(t) \tag{49}$$

where $\varepsilon(t)$ vanishes as time increases. Using Lemma 2 we conclude that $x(t)$ goes to zero.

5. EXAMPLES

We study the simple example of a system with three masses, three springs and no dashpots. The example is shown in Fig. 3.

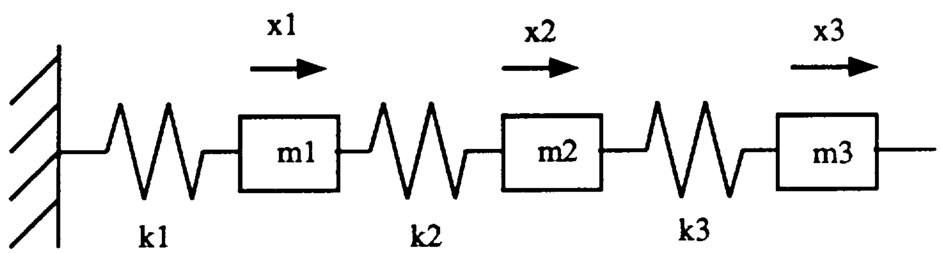

Figure 3. A Spring-Mass System

This system needs to be stabilized as it is not naturally asymptotically stable. With no control and non-zero initial conditions, the three masses oscillate since there is no damping. The equations describing the system in Fig.3 are

$$\begin{cases} m_1\ddot{x}_1 + (k_1+k_2)x_1 - k_2x_2 = u_1 \\ m_2\ddot{x}_2 - k_2x_1 + (k_2+k_3)x_2 - k_3x_3 = u_2 \\ m_3\ddot{x}_3 - k_3x_2 + k_3x_3 = u_3 \end{cases} \tag{50}$$

The matrices M, D and K are

$$M = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \tag{51}$$

$$D = 0 \tag{52}$$

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \tag{53}$$

M and K are positive definite as long as none of the masses and the spring constants is equal to zero. Several possible controller designs can be used here.

<u>m=n=l=3</u>

There are three control parameters here. A reasonable choice is

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{54}$$

and the control vector u is defined by $u^T = (u_1\ u_2\ u_3)$. Obvious solutions to Eq.(11) are given by

$$\begin{aligned} \Lambda &= I_{3\times 3} \\ H_v &= \frac{1}{2} B^T \\ B' &= \lambda B \\ M_a &= \frac{1}{\lambda} I_{3\times 3} \end{aligned} \tag{55}$$

where λ is an arbitrary strictly positive real number. As a consequence, the vector x_a is generated by the differential equation

$$\ddot{x}_a = \ddot{x} + \lambda B u \tag{56}$$

All the assumptions of Theorem 3 are satisfied. The vector x may therefore be controlled with the help of any dissipative feedback controller. A simple choice consists of taking a constant controller, that is a controller with a transfer matrix of the form k I, where I is the identity matrix.

The simulation is made on MATLAB. The following values are used in the simulation:

$$m_1 = m_2 = m_3 = 1 \tag{57}$$

$$k_1 = 1 \quad k_2 = 2 \quad k_3 = 3 \tag{58}$$

The initial conditions were arbitrarily chosen to be

$$x_{1_0} = 5 \qquad x_{2_0} = -2 \qquad x_{3_0} = 9 \tag{59}$$

$$\dot{x}_{1_0} = 3 \qquad \dot{x}_{2_0} = 5 \qquad \dot{x}_{3_0} = -4 \tag{60}$$

For the vector x_a, we choose the simple initial conditions

$$x_a = 0 \qquad \dot{x}_a = 0 \tag{61}$$

The constant λ is equal to 0.5. The gain of the feedback controller is k =1. The plot of the displacements is shown in Fig. 4. In the following plots, x_1 is indicated by —, x_2 is indicated by ··· and x_3 is indicated by ---.

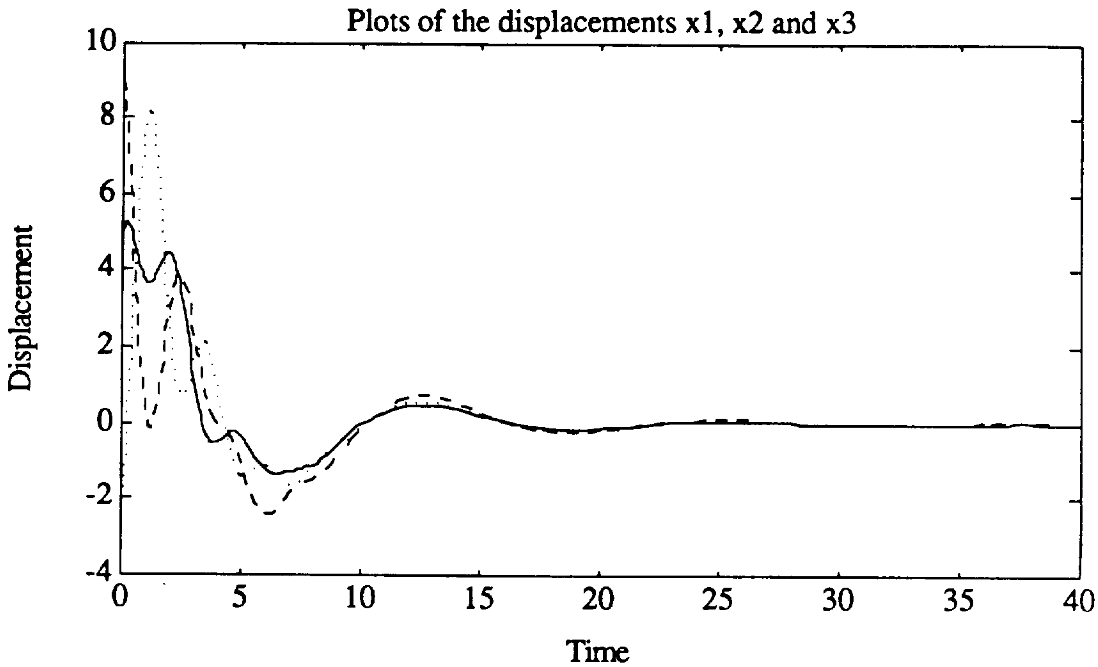

Figure 4

The control goal is achieved. The three displacements vanish with time. Nevertheless, this design requires that a force be applied on each of the masses. It is possible to reduce the number of actuators with the following control design.

<u>m=l=2</u>

Here only two forces are be applied to the system. Thus there are three possible choices, depending on what masses the forces are applied. Let us start with

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{62}$$

This choice means that the forces are applied on the masses m_1 and m_2. The control vector u is such that $u^T = (u_1 \quad u_2)$. The vector x_* is now a vector with dimension 2. Eq.(11) has the following obvious solution

$$
\begin{aligned}
&\Lambda = B^T \\
&H_v = \frac{1}{2} I_{2\times 2} \\
&B' = \lambda I_{2\times 2} \\
&M_a = \frac{1}{\lambda} I_{2\times 2}
\end{aligned}
\tag{63}
$$

where λ is an arbitrary strictly positive real number and I denotes the identity matrix. Thus x_a can be computed from the following differential equation.

$$
\ddot{x}_a = \ddot{x} + \lambda u \tag{64}
$$

and the output of the system is $y = \frac{1}{2}\dot{x}_a$.

A dissipative controller must be chosen to control the system. Here again, a constant controller is a simple possible choice. Its transfer function is k I, where k is a positive constant.

It remains to ensure that $B^T\ddot{x} = 0$ and $u = 0$ imply $x = 0$. If $B^T\ddot{x} = 0$ and u=0, then the dynamical equations of the system become

$$
\begin{cases}
(k_1 + k_2)x_1 + k_2 x_2 = 0 \\
-k_2 x_1 + (k_2 + k_3)x_2 - k_3 x_3 = 0 \\
m_3 \ddot{x}_3 - k_3 x_2 + k_3 x_3 = 0
\end{cases}
\tag{65}
$$

By differentiating the second equation and solving for $\ddot{x}_3$, we have

$$
\ddot{x}_3 = -\frac{k_3}{k_2}\ddot{x}_1 + \frac{(k_2 + k_3)}{k_3}\ddot{x}_2 \tag{66}
$$

As $\ddot{x}_1$ and $\ddot{x}_2$ are both zero, $\ddot{x}_3$ is also equal to zero. Thus the above equations are reduced to Kx = 0. Since K is positive definite, this yields x = 0. All the assumptions of Theorem 3 are satisfied and we are now assured that x will go to zero.

The closed-loop system is simulated with the same parameter choice as before. The plot of the displacements can be seen in Fig. 5. Here again the stabilization goal is achieved since the three displacements vanish as time increases.

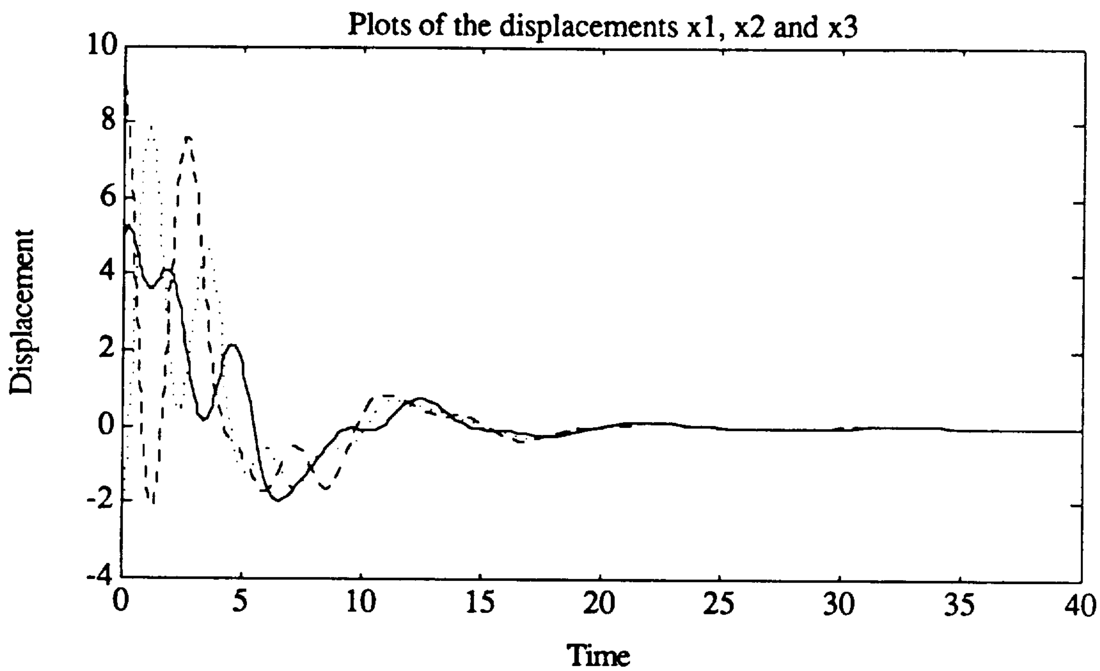

Figure 5

It is possible to stabilize this system with a different distribution of forces. For instance, let

$$B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{67}$$

Two forces are applied respectively on mass 2 and mass 3. With the same design as above, a controller for the system can be designed. The plot of the displacements is presented in Fig. 6

with the same initial conditions.

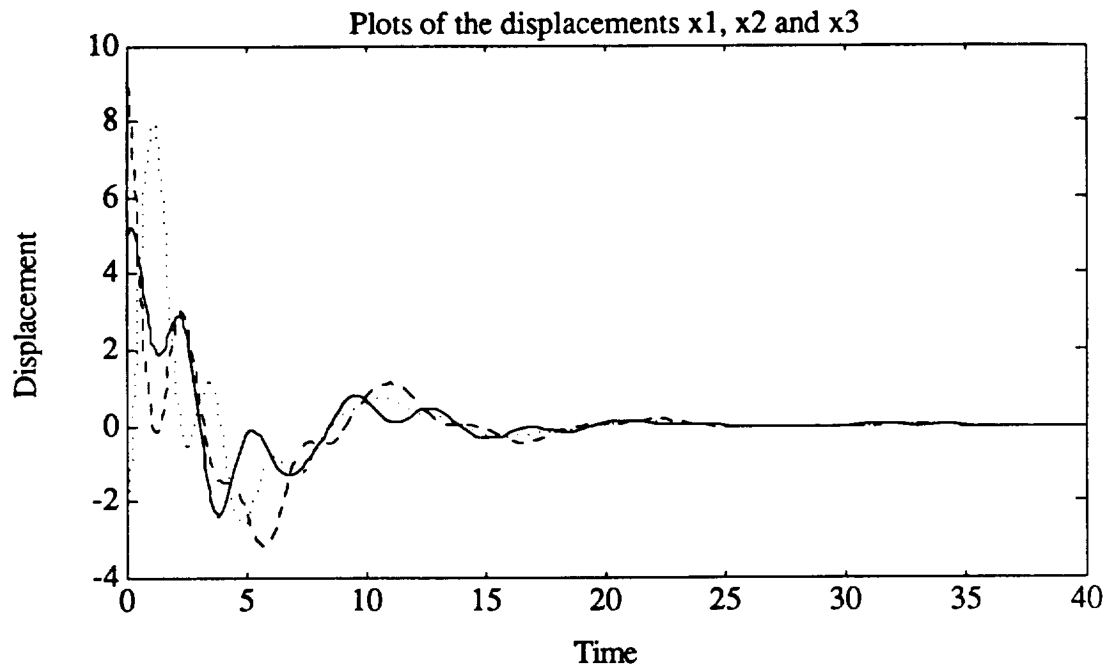

Figure 6

Finally a third possible choice is

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{68}$$

In this case some forces are applied to mass 1 and mass 3. The plot of the simulation can be seen in Fig. 7.

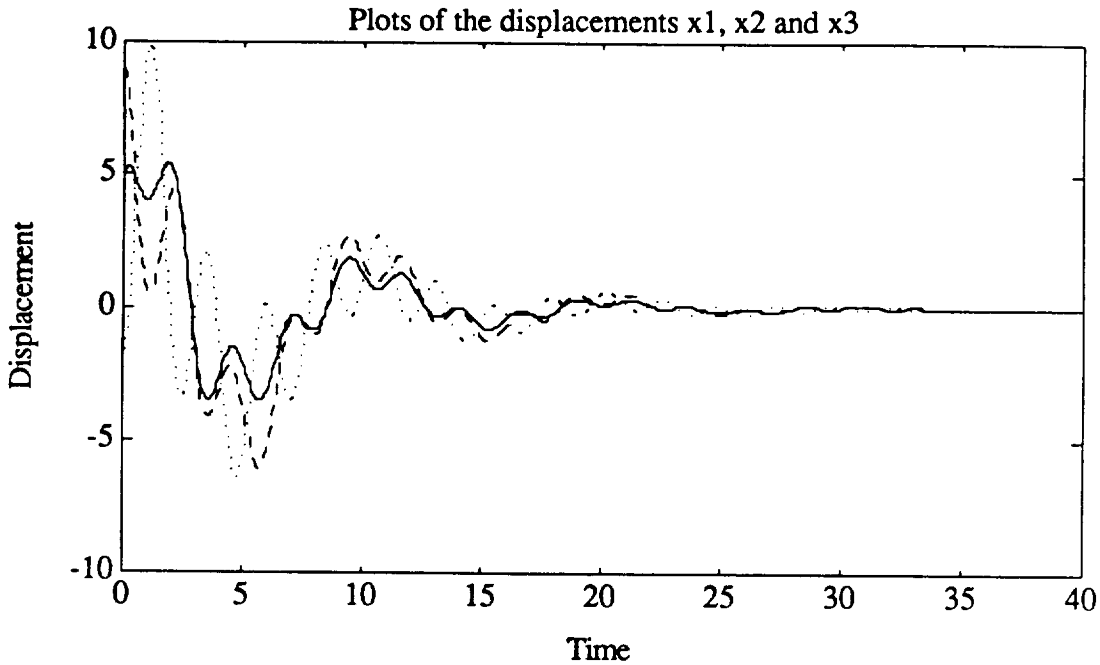

Figure 7

In all these case, the system is stabilized with the help of only two actuators.

m=l=1

Here we design a control system with only one actuator. This actuator may be located on any of the three masses. Let us first apply a force on mass 1, i.e. the matrix B is

$$B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{69}$$

Eq.(11) in Theorem 2 has the following obvious solution

$$
\begin{aligned}
\Lambda &= B^T \\
H_v &= \frac{1}{2} \\
B' &= \lambda \\
M_a &= \frac{1}{\lambda}
\end{aligned}
\tag{70}
$$

where λ is an arbitrary strictly positive real number. The state x_a is calculated by integrating the differential equation

$$
\ddot{x}_a = \ddot{x}_1 + \frac{1}{2}u \tag{71}
$$

The output of the system is $y = \frac{1}{2}\dot{x}_a$.

Here again the SPR controller is chosen to be constant. Its transfer matrix is of the form G(s) = k, where k is any strictly positive real number. With this choice we are assured that x converges to zero.

It should be checked as before that $B^T\ddot{x} = 0$ and $u = 0$ imply $\ddot{x} = 0$. The procedure is unchanged and once again those assumptions yield K x = 0. Since K is assumed to be positive semi-definite, x is necessarily equal to zero.

The simulation is run with the same choice of initial conditions. The constant λ is still equal to 0.5, and k is equal to 1. The three displacements go to zero as expected which can be seen in Fig. 8.

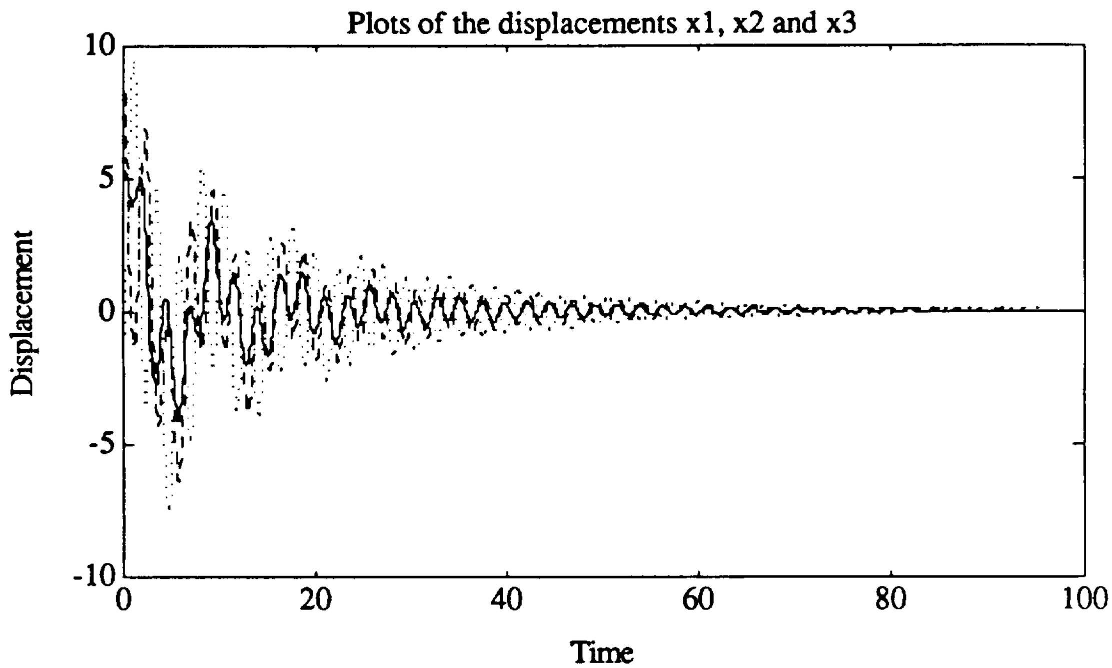

Figure 8

The force could be applied on mass 3. The matrix B for this situation is

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{72}$$

The design method is unchanged. The closed-loop system has been simulated in this case with the same initial conditions and the same choice for the parameters involved. The plot is shown in Fig. 9.

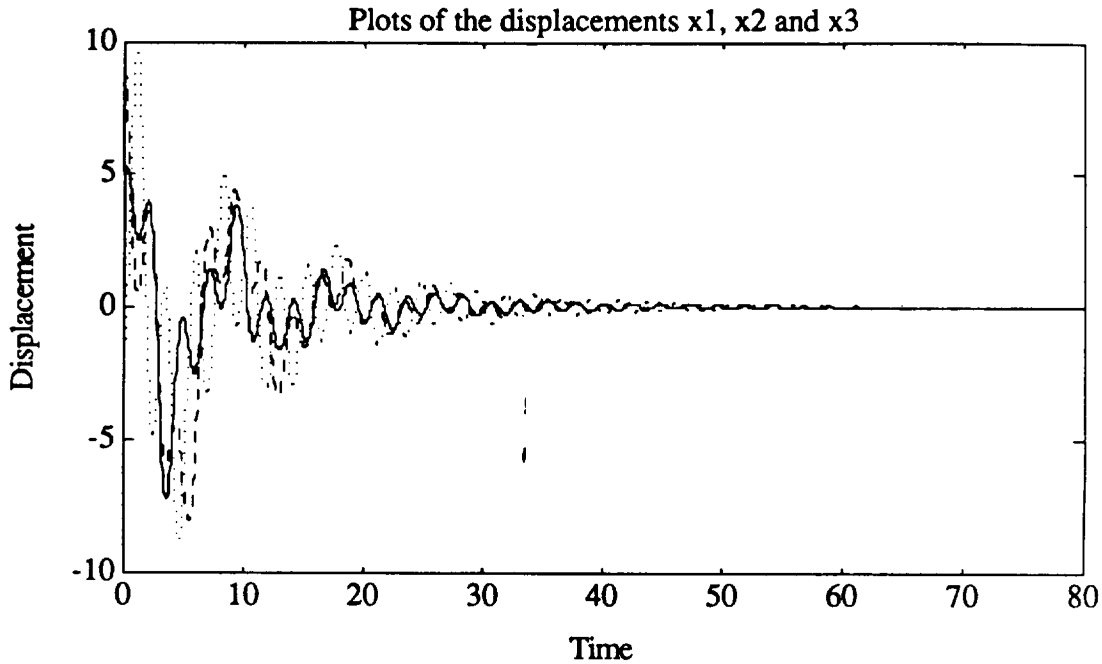

Figure 9

We could choose to apply the force on mass 2. In this case,

$$B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{73}$$

Nevertheless, it can be checked that condition (i) of Theorem 3 is not satisfied in this case. Thus no controller design can be implemented with the above choice.

6. CONCLUSIONS

The control method presented in this report is particularly of interest for practical reasons. Only acceleration at certain locations of the system need to be measured by using common accelerometers. Furthermore, the design is model independent and no knowledge of the constants of the dynamic system is required. Finally, any strictly positive real controller can be used. Thus it is possible to choose one that yields a satisfactory transient response.

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